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REPORT

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Title:
Reducing noise in radiometric multi-channel data using noise-adjusted singular value decomposition (NASVD) and maximum noise fraction (MNF).

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Summary:
The airborne gamma-ray spectrometric method is primarily used for the detection of ore bodies, fallout from nuclear accidents and as a lithological mapping tool. Modern gamma-ray spectrometers can record at least 256 channels of data in the energy range 0-3 MeV.

After applying conventional processing, there is still noise in the spectra. Several approaches are suggested to remove this remaining noise. These methods include noise-adjusted singular value decomposition (NASVD), with or without clustering, and maximum noise fraction (MNF). The methods are applied to raw spectra data prior to integrating counts in the K, U and Th windows and proceeding to conventional processing.

The three methods are applied to a survey data set for comparison. All methods are shown to reduce noise in the original data set.

Keywords: Geofysikk	Radiometri	Spektrometri
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CONTENTS

1. INTRODUCTION

The airborne gamma-ray spectrometric method is primarily used for the detection of ore bodies, fallout from nuclear accidents, radon hazard and as a lithological mapping tool. Modern gamma-ray spectrometers can record at least 256 channels of data in the energy range 0-3 MeV. These data are summed into three energy windows characteristic of potassium, uranium and thorium. The windows are then corrected for instrument livetime, and background contributions from aircraft radioactivity, cosmic radiation and atmospheric radon are removed. The windows are then subject to spectral stripping and conversion of count rates to apparent radioelement concentrations (IAEA 1991, Geosoft 2000).

After applying this processing, there is still noise in the spectra. Several approaches are suggested to remove this remaining noise. These methods include noise-adjusted singular value decomposition (NASVD) with or without clustering (Hovgaard 1997, Hovgaard & Grasty 1997, Minty & McFadden 1998) and maximum noise fraction (MNF) (Green et al. 1988, Dickson & Taylor 1998, 1999 & 2000). The methods are applied to raw spectra data prior to integrating counts in the K, U and Th windows and proceeding to conventional processing.

This report outlines these methods and presents an example where the different methods are employed.

2. NOISE-REDUCTION METHODS

2.1 Noise-adjusted singular value decomposition (NASVD)

Noise-adjusted singular value decomposition (NASVD) is a method for removing noise from raw gamma-ray spectra using spectral component analysis. The method transforms the observed spectra into orthogonal spectral components. The lower-order components are interpreted to represent signal, and the higher-order components are interpreted to represent noise. The component spectra that represent the noise are rejected, and the original spectra are 'smoothed' by reconstructing the spectra from the lower-order components only.

The NASVD method uses a multivariate statistical analysis to extract the dominant spectral shapes from a series of raw input spectra. The most effective way to do this is to use orthogonal components. The method is similar to principal component analysis (PCA), which is widely used to analyse multivariate systems.

The principal components of a set of m spectra $\mathbf{A}$ (m x 256, where $m \geq 256$) are the eigenvectors of the covariance matrix $\mathbf{A}^T\mathbf{A}$. The principal components are mutually

orthogonal and are sorted by eigenvalue into descending order. The first principal component is the average spectrum shape for the input spectra. If the first component contribution were subtracted from each of the input spectra, then the second principal component would be the average shape of the resultant differences, and so on for subsequent components. The eigenvalues are the variances associated with each principal component. The observed spectra can then be represented as a linear combination of the principal components;

$$\mathbf{A=CS} \quad (1)$$

A = input spectra (m x 256)
C = concentrations (m x 256)
S = principal components (256 x 256)

Since the lower order components represent the signal in the original observed spectra, and the higher-order components represent uncorrelated noise, the noise can be removed by reconstructing 'smoothed' spectra from lower-order components only.

There are two constraints associated with the use of the standard principal component method for multi-channel spectra. First, the variance associated with each channel must be the same. Second, the principal component method requires the data to be mean centred. The second problem is overcome by using the singular value decomposition (SVD) method to analyse the dispersion of the data around the origin rather than the sample mean. This produces eigenvectors, which are used in the same way as the principal components in a principal component analysis.

Hovgaard (1997) suggested a simple solution to the first problem. The best fit of the mean spectrum to each input spectrum provides a good estimate of the mean count rate (and variance) for each channel. **A** in (1) is noise adjusted by scaling each observed spectrum by the best fit of the mean spectrum to each input spectrum;

Sum spectrum: $Sum(i) = \sum_{j=1}^{n} A(i, j)$ $\quad , n = \#\, of\ spectra,\ j = channel\#, i = spectrum\#$

Normalised spectrum: $S(j) = \dfrac{\sum_{k=1}^{n} A(i,k)}{Sum(i)}$

Normalised unit spectrum: $SN(j) = \dfrac{S(j)}{\sum_{k=1}^{n} S(k)}$

Noise-adjusted spectrum: $A_{new}(i, j) = \dfrac{A_{old}(i, j)}{\sqrt{SN(j) * Sum(i)}}$

The $\mathbf{A}_{\mathbf{new}}$ is input to the singular value decomposition routine. The noise-adjusted spectrum can be written as;

$$\mathbf{A}=\mathbf{U}\mathbf{W}\mathbf{V}^{\mathbf{T}} \quad (2)$$

where **U** is i • j (i= number of spectra, j = number of channels) and **W** is a j • j diagonal matrix. The eigenvectors of $\mathbf{A}^{\mathbf{T}}\mathbf{A}$ are **V**, and the eigenvalues are the square of the elements of **W**. Equation (2) is used to calculate the eigenvectors (**V**) and amplitudes (**UW**) of **A**. The eigenvectors are rescaled by multiplying them by the normalised unit spectrum, and the amplitudes are rescaled by multiplying by the sum of the counts in each input spectrum. The first components of **W** and **V** contain most of the signal. The noise is removed by reconstructing spectra from the lower order eigenvectors and amplitudes (normally, 6-8 components are sufficient to reconstruct the signal).

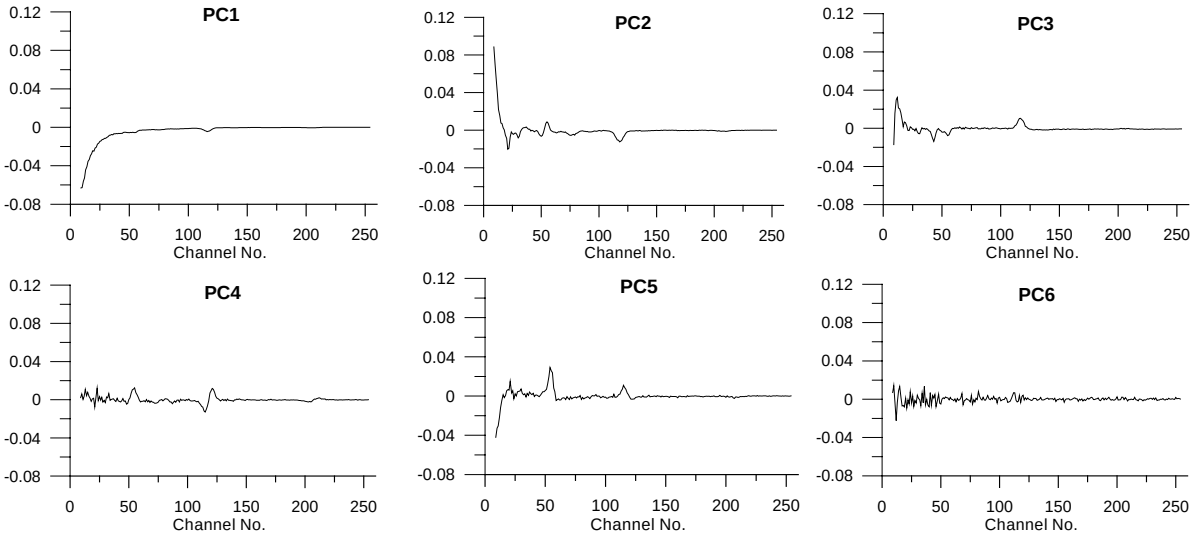

Fig. 1. First 6 principal components for a survey.

Fig. 1 shows the first 6 principal components for a survey data set. The first component is the average spectrum. Components 2-5 show a signal at channel 55 from Cs^{137}. A signal between channels 109-125 is due to K (potassium). The components are slightly shifted due to instrument energy drift. Principal component 4 shows signal in the thorium window between channels 189 and 220. Channel 6 is dominated by incoherent spectral shapes and represents mostly noise. Hence, 5 components are probably sufficient to reconstruct the signal in this case.

2.2 NASVD with clustering

Tests on synthetic data suggest that the smaller the variation in spectral shape within the input signal, the greater the noise reduction (Minty & McFadden 1998). The entire survey database is sorted into clusters on the basis of similarity in spectral shape, and the NASVD method is applied individually to each of these clusters. This reduces the K, U and Th fractional errors compared with those obtained when data are processed without clustering. Due to the similarity of spectra in each cluster, fewer components are needed to reconstruct the signal.

The survey spectra are sorted as follows. The spectra are smoothed by applying the NASVD method to either the entire survey or individual flights. Shape factors are calculated for each spectrum. These shape factors are smoothed, either by low-pass filtering or median filtering. These filtered shape factors are used to sort the original spectra into clusters using the migrating means algorithm (Richards 1993). Using this algorithm, the spectra are initially randomly assigned to a fixed number of clusters. Spectra are then iteratively moved from one cluster to another in such a way that the sum of the distances between spectra and their cluster means is minimised. For a survey comprising 100000 spectra, 10 clusters are used to give about 10000 spectra per cluster. NASVD smoothing is then applied to each cluster of spectra. This requires less principal components for spectral reconstruction. The shape factors are determined according to Fig. 2 (after Minty & McFadden 1998). The shape factors are explained below;

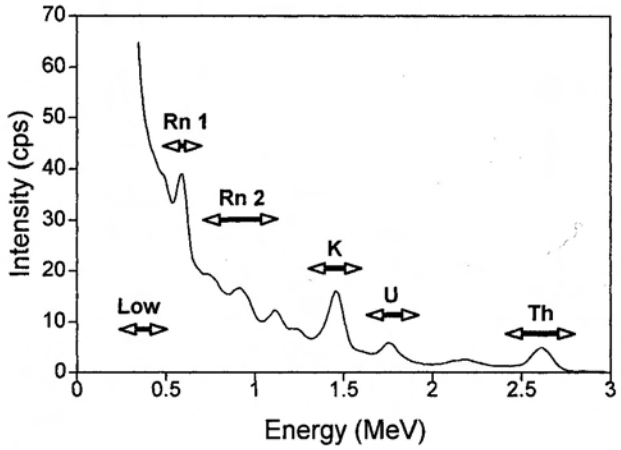

Fig. 2. Definition of shape factors (after Minty & McFadden 1998).

1) The contribution of atmospheric radon to the observed spectra (ratio of window centred on 0.61 MeV and a broader window at a slightly higher energy).
2) The ratio of the contributions of Th and K sources to the observed spectra (Th window counts / K window counts).
3) The ratio of the contributions of Th and U sources to the observed spectra (Th window counts / U window counts).
4) A height factor describing the change in spectrum form with distance between the source and detector (Low energy window / (K window + U + Th)).

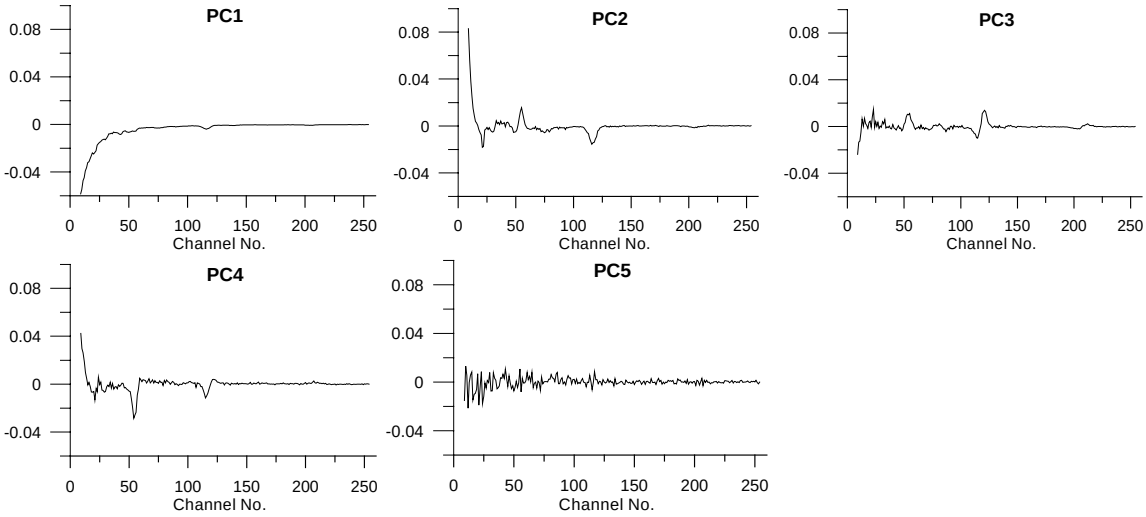

Fig. 3. First 5 principal components for a survey.

Fig. 3 shows the first five components of a cluster processed with this method. Note that only the first four components show coherent spectral shapes, so only four components are needed to reconstruct the signal, thus making noise removal more prominent.

2.3 Maximum noise fraction (MNF)

This method is routinely used for improving image quality in multi-band data. One of the most common measures of image quality is the signal-to-noise ratio. Instead of choosing new components to maximise variance, as the principal components transform does, we now choose to maximise the signal-to-noise ratio (Green et al. 1988).

Let us consider a multivariate data set of p channels;

$\mathbf{Z_i}(x)$, i=1..p, where x represents spectra.

We assume that $\mathbf{Z}(x)=\mathbf{S}(x)+\mathbf{N}(x)$

$\mathbf{S}(x)$ and $\mathbf{N}(x)$ are the uncorrelated signal and noise components of $\mathbf{Z}(x)$. Thus;

$$\mathrm{Cov}\{\mathbf{Z}(x)\}=\mathbf{\Sigma}=\mathbf{\Sigma_S}+\mathbf{\Sigma_N}$$

where $\mathbf{\Sigma_S}$ and $\mathbf{\Sigma_N}$ are the covariance matrices of $\mathbf{S}(x)$ and $\mathbf{N}(x)$, respectively.

The noise fraction of the i^{th} channel is

$$\mathrm{Var}\{\mathbf{N_i}(x)\}/\mathrm{Var}\{\mathbf{Z_i}(x)\},$$

the ratio of noise variance to the total variance for that channel. The maximum noise fraction (MNF) transform chooses linear transformations;

$$\mathbf{Y_i}(x)=\mathbf{a_i}^{\mathbf{T}}\mathbf{Z}(x),\ i=1..p$$

such that the noise fraction for $\mathbf{Y_i}(x)$ is maximum along all linear transformations orthogonal along all linear transformations orthogonal to (x) is maximum along all linear transformations orthogonal along all linear transformations orthogonal to $\mathbf{Y_j}(x)$, j=1..i.

It can be shown that the vectors $\mathbf{a_i}$ are the left-hand eigenvectors of $\mathbf{\Sigma_N}\ \mathbf{\Sigma}^{-1}$ and that λ_i, the eigenvalue corresponding to $\mathbf{a_i}$ equals the noise fraction in $\mathbf{Y_i}(x)$. $\lambda_1 \geq \lambda_2 \geq .. \geq \lambda_p$ will show increasing signal quality.

An important property of the MNF transform (not shared by principal components) is that, because it depends on signal-to-noise ratios, it is invariant under scale changes to any channel. To obtain the MNF transform, we need to know both $\mathbf{\Sigma_N}$ and $\mathbf{\Sigma}$. Usually $\mathbf{\Sigma}$ can be estimated by using the difference between adjacent spectra (Green et al. 1988, Dickson & Taylor 1998, 2000).

3. NASVD vs. MNF

There has been a debate about which of the two methods NASVD and MNF works best at differentiating between the geological signal and noise. While it is argued that the MNF method is better (Dickson & Taylor 1998, 1999, 2000) at noise removal, Hovgaard & Grasty (1998) and Minty (1998) points to a few problems involved using the MNF method;

1) Short wavelength signal is interpreted as noise.
2) The method finds too many principal components for gamma-ray spectra.
3) The MNF method assumes the noise level is the same on one particular channel throughout the survey area.

A problem with NASVD is that it does not necessarily transfer all of the noise to the rejected principal components. Survey segments having disproportionate local variance in particular markedly disturb the retained eigenvectors. In short, MNF seems to isolate noise better than the NASVD method.

According to Dickson & Taylor (1999), it seems that where noise reduction is required, the method should be optimised to isolate noise. This the MNF procedure does by using a measure of signal-to-noise ratio. If, however, one is interested in isolating different isotopes by obtaining the least number of spectral shapes that explain the data variance, the NASVD may be considered. MNF cannot be applied to this task.

4. CASE HISTORY

We test the noise reduction methods using airborne gamma ray spectrometry data from a helicopter-borne geophysical survey flown by the Geological Survey of Norway for Crew Development Corporation over the island of Ringvassøy in the county of Troms, Norway (Mogaard & Mauring 2002). The survey covers approximately 320 square kilometres of mountainous terrain. The survey aircraft was an Areaspatiale Ecureuil SA350 B-3. Flying speed was approximately 100 km per hour (28 meters per second). Flight lines over the survey area were in directions North and South with a flight line spacing of 200 m. The radiometric sensors were mounted immediately beneath the helicopter. The radiometric system consists of four sodium iodide (NaI) crystals having a total volume of 1024 cubic inches (16.78 litres). The NaI crystals are coupled to a 256 channel gamma ray spectrometer. Registration rate is one per second. An upward looking crystal was used in this survey, and can if desired be used to correct for airborne radon contamination.

The spectrometer is an energy pulse height analyser that sorts data into 256 channels according to energy magnitude. Each channel is 0.012 MeV wide. The full 256 channel spectrum was recorded. Windows constructed from selected groups of channels record the contributions of K^{40}, Bi^{214} (a daughter product of U^{238}) and Tl^{208} (a daughter product of Th^{232}). A fourth window – the total count window – records gamma rays with energies between 0.4 MeV and 3 MeV. Further windows include the "cosmic" window, recording photon energies of >=3 MeV and a second Bi^{214} window from the upward looking detector.

These were processed using NASVD, clustered NASVD and MNF techniques using 6 components. As a measure of noise reduction, windowed (K, U and Th) data were filtered using a 7 point running median, and the mean of the absolute value was calculated between the windowed data and the filtered windowed data. These difference values are an indicator of the noise reduction of the applied processing method. Results are listed in Table 1.

Element	Method	Difference, 6 components
K	No processing	4.40 ± 3.94
	NASVD	3.73 ± 3.36
	NASVD cluster	3.47 ± 3.05
	MNF	3.67 ± 3.30
U	No processing	1.89 ± 1.52
	NASVD	0.82 ± 0.67
	NASVD cluster	0.85 ± 0.69
	MNF	0.83 ± 0.68
Th	No processing	1.93 ± 1.60
	NASVD	0.97 ± 0.81
	NASVD cluster	0.92 ± 0.82
	MNF	0.91 ± 0.74

Table 1: Difference between windowed data and windowed data filtered with a 7 point running mean.

Fig. 4 shows grids of uranium prior to any background removal and stripping for raw data (upper left), NASVD processed data (upper right), clustered NASVD processed data (lower left) and MNF processed data (lower right). NASVD- and MNF-processed spectra were reconstructed using six principal components and 10 clusters.

Fig. 4. Grids of uranium window count rate prior to background removal and window stripping (upper left), NASVD processed data (upper right), clustered NASVD processed data (lower left) and MNF processed data (lower right). The mapped area is approximately 32 km across.

Fig. 4 shows that the NASVD, clustered NASVD and MNF methods all result in a significant reduction in noise in the Ringvassøy data set. All methods worked equally well on this particular data set. Clustering clearly had little affect on the result of NASVD processing (compare Fig. 4 top right and bottom left). This is most likely because the surveyed region is small and the geology and related spectral characteristics vary little from one place to another. The MNF processed data differ slightly from the NASVD data sets, but it is difficult to say which is better.

5. CONCLUSIONS

Three methods for noise reduction of multi-channel radiometric data are described in this report. These are noise-adjusted singular value decomposition (NASVD), NASVD with clustering and maximum noise fraction (MNF). All methods are shown to reduce noise in the original data set.

6. ACKNOWLEDGEMENT

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