Permeability development during hydrofracture propagation in layered reservoirs

SONJA L. BRENNER & AGUST GUDMUNDSSON

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Hydrofractures are generated by internal fluid overpressure and together with shear fractures, contribute significantly to the permeability of water, oil or magma reservoirs. We use boundary-element models to explore the effects of abrupt changes in layer stiffness on the propagation, arrest and aperture variation of hydrofractures. The results of numerical models and field observations indicate that changes in Young's moduli can contribute to the arrest of hydrofractures. When the fluid overpressure is the only loading, hydrofractures are more likely to propagate through stiff layers (if they are not stress barriers) than through soft layers. If most hydrofractures become arrested because of mechanical layering in a heterogeneous fluid reservoir, the associated fracture system will be poorly interconnected and thus of low permeability. Aperture variation is of great importance in bedrock hydrogeology, particularly because channelling of the fluid flow along the widest parts of a fracture may occur. When the fluid overpressure of the hydrofracture is the only loading, the hydrofracture aperture tends to be greater in soft layers than in stiff layers. These results suggest that aperture variations may encourage preferential flow (flow channelling) in layered reservoirs.

Sonja Brenner & Agust Gudmundsson. Geological Institute, University of Bergen, Allegaten 41, N-5007 Bergen, Norway (Sonja.Brenner@geol.uib.no; Agust.Gudmundsson@geol.uib.no).

Introduction

Hydrofractures (fluid-driven fractures) are partly or wholly generated by internal fluid overpressure (net pressure or driving pressure). They are commonly extension fractures. Examples include dykes, mineral-filled veins and man-made hydraulic fractures as well as many joints. Hydrofractures, together with shear fractures, contribute significantly to the permeability of heterogeneous fluid reservoirs, whether the fluid is water, oil or magma. Although heterogeneities occur at various scales in reservoirs, for the propagation of outcrop-scale hydrofractures, as are discussed here, perhaps the most important heterogeneity in reservoirs is mechanical layering (Economides & Nolte 2000, Gudmundsson & Brenner 2001).

The linking up of discontinuities during hydrofracture propagation is likely to be one of the main mechanisms for generating and maintaining permeability in heterogeneous reservoirs. Another mechanism for generating permeability in reservoirs, the formation of shear fractures (faults) through the linking up of small fractures, has been studied extensively in recent years (e.g., Cox & Scholz 1988, Cartwright et al. 1995, Acocella et al. 2000, Mansfield & Cartwright 2001).

To explore the conditions for propagation of natural hydrofractures is important for groundwater exploration, as well as for the use of geothermal energy and petroleum. In petroleum engineering, the aim is that the hydraulic fracture propagates only along the target layer (the reservoir) to increase its permeability. Thus, the hydraulic fracture should be confined to the target layer and be arrested at the contacts with the layers above and below. Natural, arrested hydrofractures that are confined to single layers with non-fractured layers in between, however, contribute significantly less to the overall permeability of a reservoir than do fractures that propagate through many layers. This follows because only interconnected fracture systems reach the percolation threshold (Stauffer & Aharony 1994).

Once an open fracture has formed, for example a joint, its permeability depends much on its aperture. In particular, because channelling of the fluid flow along the widest parts of a fracture may occur (Tsang & Neretnieks 1998), aperture variation is of great importance in bedrock hydrogeology. The aperture depends, among other parameters, on the mechanical properties of the host rocks. Thus, in a layered reservoir, the mechanical differences between the layers are likely to affect the size of the aperture, even for a fracture with constant fluid overpressure.

In this paper, we first summarise the basic concepts of permeability of fractured rocks and hydrofracture propagation. Secondly, we use boundary-element models to explore the conditions for hydrofracture propagation and their aperture variation, focusing on the effects of changes in mechanical properties of the host rock. We then compare these results with field observations and discuss the implications for permeability generation in fluid-filled, layered reservoirs.

Permeability and fluid flow in reservoirs

In sediments, there is a positive correlation between the permeability and the (primary) porosity. In solid rocks this correlation does normally not hold. One reason for this is that diagenetic processes such as compaction and cementation reduce the effective porosity, i.e., the interconnected pore space that is available for fluid flow. Another reason, which applies also to igneous and metamorphic rocks, is that most of the fluid flow in solid rocks occurs through fractures that form a secondary porosity. In the case of an impermeable host rock, all fluids would flow through interconnected open fractures. In contrast to fluid flow in porous media, fluid flow in fractured media is still not well understood (Singhal & Gupta 1999).

The volumetric flow rate (laminar flow) through an isolated fracture with smooth, parallel, fracture walls is proportional to the cube of the aperture of the fracture (the cubic law). From this it follows that small changes in a fracture aperture may lead to great changes in its fluid transport. Also, if the fracture has rough walls, or the fracture aperture varies much along the trace of the fracture, channelling of the fluid flow along the widest parts of the fracture may be important (Tsang & Neretnieks 1998).

The cubic law for single fractures can be extended to fracture sets (Bear 1993). For example, the volumetric flow rate through a set of parallel fractures in near-surface conditions, and away from large fault zones, can be calculated based on the fracture opening and the distance between the fractures (Singhal & Gupta 1999).

The permeability of a fractured reservoir depends on the connectivity of its fracture systems. If a fluid can flow through the whole reservoir, using an interconnected system of open fractures, its percolation threshold is reached (Stauffer & Aharony 1994). The current stress field also controls fluid flow in fractured reservoirs (Faybishenko et al. 2000, Gudmundsson 2000). Fractures are sensitive to changes in the stress field and deform much more easily than circular pores. In addition, the stress field contributes to the fluid overpressure. Overpressured fluids probably develop many interconnected fracture systems in reservoirs through the propagation of hydrofractures; but arrested hydrofractures crossing only one or a few layers cannot contribute much to the overall permeability of a reservoir.

Hydrofractures

The growth of a hydrofracture depends primarily on the mechanical properties of the host rock and the fluid overpressure of the fracture. Fluid overpressure is defined as the total fluid pressure minus the stress normal to the fracture. For extension fractures, modelled as mode I (opening mode) cracks, as is appropriate for most hydrofractures (Gudmundsson et al. 2001), this is the fluid pressure in excess of the minimum principal compressive stress (maximum principal tensile stress).

To solve problems in rock mechanics, at least two elastic moduli must be determined. The moduli most commonly used are Young's modulus and Poisson's ratio (Hudson & Harrison 1997). Poisson's ratio is a measure of the absolute ratio of strain in perpendicular directions; 0.25 is common for many solid rocks (Jaeger & Cook 1979, Jumikis 1979). Young's modulus is a measure of the stiffness of a linear elastic material, which is approximately the behaviour of many rocks up to 1-3 % strain at low temperature and pressure (Paterson 1978, Farmer 1983). For these, stress varies linearly with strain (Hooke's law) and the ratio of stress and strain is the rock's Young's modulus. The stiffnesses of common rock types range from very soft, $E < 1$ GPa, for example mudstone, to very stiff, $E > 100$ GPa, for some crystalline rocks (Bell 2000). Because hydrofracture propagation is normally slow compared with the velocity of seismic waves, static Young's moduli (rather than dynamic) are appropriate and used in the models below. In these models we use the laboratory values of the Young's moduli, which may be 1.5-5 times greater than the in situ values of the same rock types (Heuze 1980). The highest Young's moduli thus yield somewhat higher stresses than would occur, for the same loading conditions, in nature. The implications of the numerical models for the general fracture geometries and stresses would, however, not be much different if lower Young's moduli were used.

Hydrofractures are initiated when the fluid pressure exceeds the minimum principal compressive stress by the tensile strength of the host rock. Typical in situ tensile strengths of rocks are in the order of 0.5-6 MPa (Haimson & Rummel 1982, Schultz 1995, Amadei & Stephansson 1997). The propagation is made possible by the linking up of discontinuities in the host rock ahead of the hydrofracture tip. Discontinuities are significant mechanical breaks in the rock, normally with low or negligible tensile strengths.

Analytical models of hydrofractures in homogeneous, isotropic rocks show that the theoretical tip tensile stresses are normally very high so that a continuous and buoyant hydrofracture should continue its vertical propagation to the surface (Gudmundsson & Brenner 2001). For example, in a mathematical crack subject to constant fluid overpressure, the theoretical tensile stresses at its tips approach infinity. Similarly, linearly varying overpressure distribution in a hydrofracture gives infinite crack-tip tensile stresses. Infinite stresses, however, do not occur in rocks because fracture-tip cracking and plastic flow lowers the stresses. Nevertheless, for hydrofractures in homogeneous, isotropic rocks, very high crack-tip tensile stresses are expected; and these stresses by far exceed common tensile strengths of rocks (Gudmundsson & Brenner 2001).

Analytical models also indicate that, in a homogeneous isotropic rock, a fracture subject to constant fluid overpressure opens up into a hydrofracture with a smoothly varying, elliptical aperture (Sneddon & Lowengrub 1969, Gudmundsson 2000, Maugis 2000). A hydrofracture subject to a linearly varying fluid overpressure has a similar shape. In heterogeneous and anisotropic rocks, however, a greater aperture variation is expected.

Numerical models

Numerical models are often used to simulate physical processes, for example when analytical solutions cannot be found or, if found, are too complex to be of practical use. Analytical solutions are not appropriate for most realistic problems concerning layered reservoirs. This applies, in particular, to problems concerning the arrest and aperture variation of hydrofractures, both of which are likely to depend, in a complex way, on the mechanical contrast between the different layers.

Most numerical programs in solid mechanics are based on the linear elasticity theory. The most commonly used programs in rock mechanics and engineering are based on the finite element method (FEM) (Zienkiewicz 1977), where the problem domain must be divided into volumetric elements for which properties are defined and solutions calculated. There is, however, an increasing use of the boundary element method (BEM) (Brebbia & Dominguez 1989), where only the surfaces need to be discretised into elements for calculation and zones with uniform properties are defined. Because exact values are calculated at the boundaries, and not extrapolated from the inside of volumetric elements like in the FEM, the BEM gives more accurate solutions for boundary problems (e.g., surface stresses). To obtain information on areas inside of the zones of the defined problem, for example the stress concentration around a fracture tip, solutions for internal points are calculated and plotted.

We use the software BEASY (1991), which is based on the BEM, to explore the effects of abrupt changes in layer stiffness on the propagation, arrest and aperture variation of hydrofractures. Abrupt changes are common in layered reservoirs. For example, layers with relatively high Young's moduli like limestone, sandstone, basalt and gneiss may alternate with softer layers such as marl, shale, tuff and amphibolite. In the models, extreme differences of mechanical properties of layers were used to emphasise the effects of interest.

The first models (Figs. 1-3) show how layers with greatly different Young's moduli influence the stress field around the tip of a hydrofracture as well as the shape of the hydrofracture itself. The models are of unit length and height and are fastened in the lower corners to avoid rigid body translation and rotation. Poisson's ratio is 0.25 in all the layers in all the models. In each model, the hydrofracture is subject to a fluid overpressure with a linear variation from 10 MPa at the bottom (the fracture centre) to 0 MPa at the tip; it is the only loading. The thick layer hosting the hydrofracture C and the surface layer A have the same stiffness, 40 GPa, in all models. This is a typical value for common rocks (Jumikis 1979, Bell 2000). The stiffness of layer B changes between model runs.

In the first model (Fig. 2), a hydrofracture is approaching a thin layer B of a high stiffness, 100 GPa. The tensile stress near the tip is much greater than the tensile strength of common rocks and would thus lead to a further propaga-

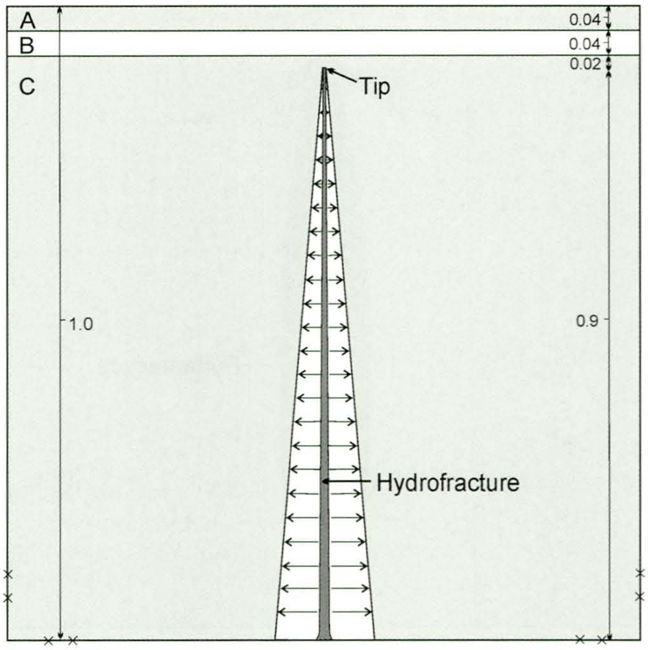

Fig. 1. Basic boundary-element configuration used for the models in Figs. 2 and 3. Each model has unit dimensions, a uniform Poisson's ratio of 0.25, and is fastened in its lower corners. The fluid overpressure in the hydrofracture varies linearly from 10 MPa at the fracture bottom (centre) to 0 MPa at the fracture tip, as indicated by horizontal arrows. The main layer, C, hosts the hydrofracture, is 0.92 units thick, and is moderately stiff (40 GPa). The two, thin, top layers are both 0.04 units thick. The uppermost layer A has the same Young's modulus as layer C in all the models. The stiffness of layer B varies between model runs.

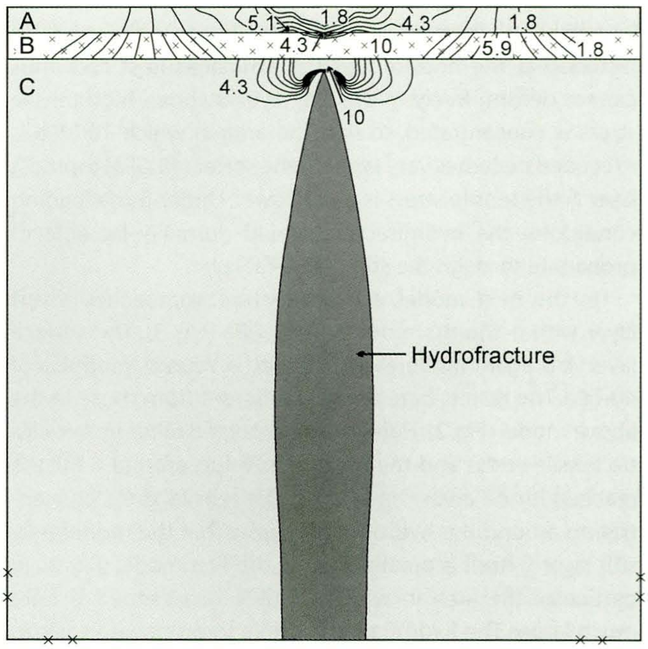

Fig. 2. The tip of a hydrofracture, located in the moderately stiff layer C, approaches a very stiff layer B (100 GPa) near to the free surface. The contours show the maximum principal tensile stress s3 in megapascals (truncated at 1 MPa and 10 MPa in all the models). The tensile stress concentration around the hydrofracture tip is very high but occurs in a rather small area. High tensile stresses are concentrated in the stiff layer B, and at the sharp fracture tip.

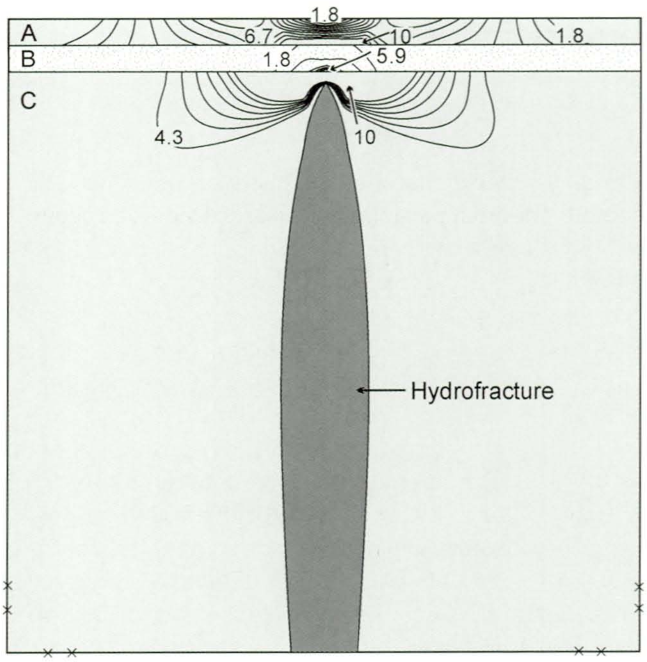

Fig. 3. Same model as in Fig. 2, except that layer B is now soft (5 GPa). Little tensile stress is transferred into this soft layer. The area with high tensile stress concentration around the hydrofracture tip inside the moderately stiff layer C is larger than in the model with the stiff layer B (Fig. 2). The tensile stress concentration in the topmost layer A, with a moderate stiffness (40 GPa), is lower, and the fracture tip is there more rounded, than in the previous model (Fig. 2).

tion of the hydrofracture tip. The tip is thin and sharp and the fracture aperture increases downwards. The largest aperture, however, does not occur in the centre of the hydrofracture (bottom of the model) because the model is fastened at the bottom and the simulated host rock thus cannot deform freely. In the stiff layer B above, high tensile stress is concentrated, so that the area in which 10 MPa is exceeded becomes very large. In the softer (40 GPa) topmost layer A the tensile stress is much lower. Under these loading conditions, the hydrofracture would normally be able to propagate through the stiff (100 GPa) layer.

In the next model, a hydrofracture approaches a soft layer with a Young's modulus of 5 GPa (Fig. 3). The surface layer A is again moderately stiff with a Young's modulus of 40 GPa. The results here are very different from those in the above model (Fig. 2). Here the lower layer B takes up very little tensile stress and the maximum value, around 6 MPa, is reached inside a very small area. The tensile stress concentration around the hydrofracture tip within the moderately stiff layer C itself is greater than in the first model (Fig. 2); in particular, the area in which 10 MPa is exceeded is here much larger. The hydrofracture is able to propagate so as to reach the contact with the soft layer B. But without a favourably orientated discontinuity in the soft layer that could open up and be used as a pathway for the hydrofracture, it is unlikely that its propagation would continue through the soft layer. The tip of the hydrofracture is more rounded than in Figure 2. In the stiff surface layer A, high tensile stresses are generated, but they are still lower than those generated in the stiff layer B of the first model (Fig. 2).

The next model focuses on the influence of layers with different stiffnesses on the aperture of a hydrofracture. The model (Figs. 4-5) is of unit height and with a Poisson's ratio of 0.25. Here we used ten layers of equal thickness (each 0.09 units). The lowermost layer J is very stiff (100 GPa), the next layer above I is very soft (1 GPa), whereas the third layer H is moderately stiff (10 GPa). This three-layer sequence is

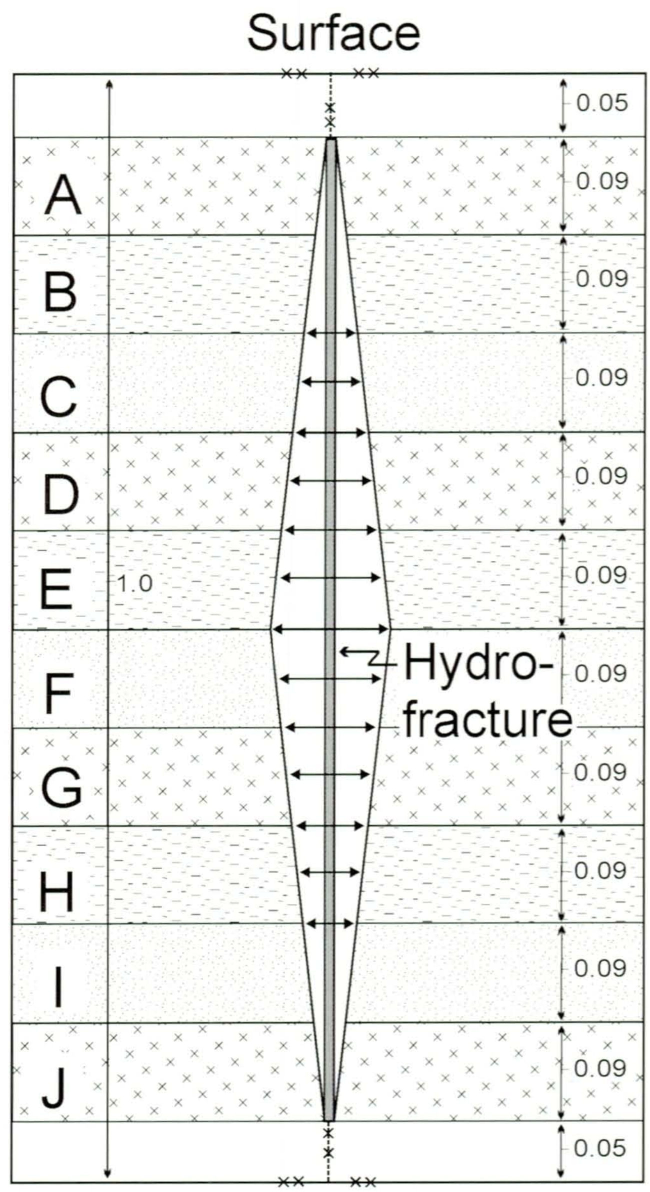

Fig. 4. Basic boundary-element configuration used for the model in Fig. 5. The fluid overpressure in the hydrofracture, indicated by horizontal arrows, is the only loading and varies linearly from 10 MPa at the fracture centre (located at the contact between layers E and F) to 0 MPa at the upper and lower fracture tips. The model has unit height (vertical dimension); all the 10 layers, A-J, have the same thickness (0.09 units) and a uniform Poisson's ratio of 0.25. The thin layers at the top and bottom of the model are used to fasten it (as indicated by crosses) and to confine the hydrofracture. A succession of 3 layers – stiff (100 GPa), soft (1 GPa), and moderate (10 GPa) – is repeated to the top, so that layer A has a Young's modulus of 100 GPa.

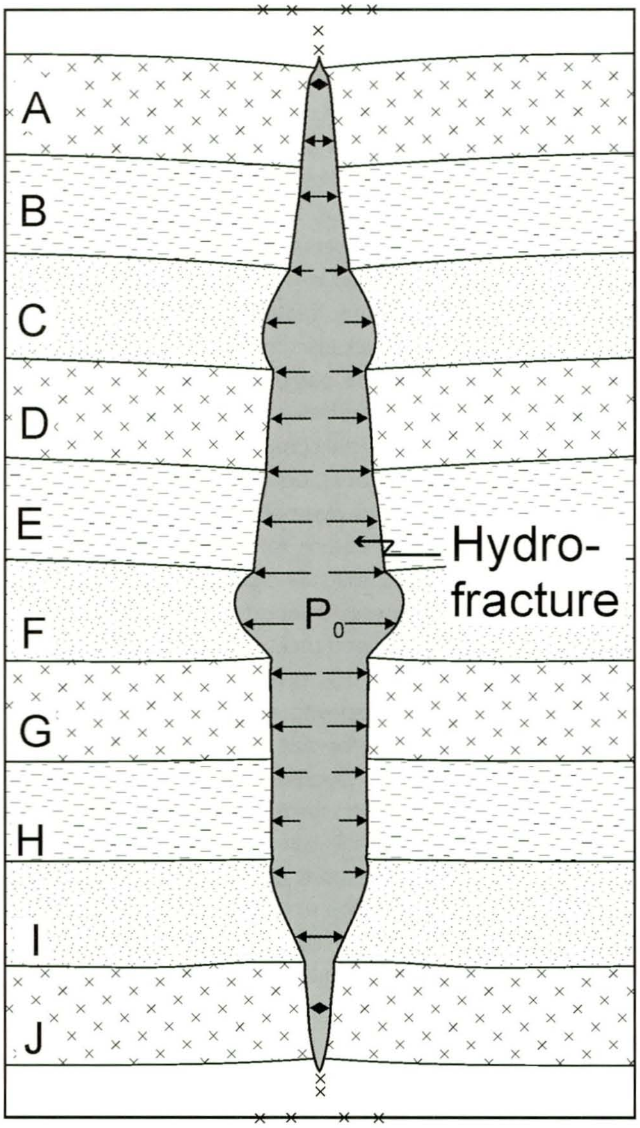

Fig. 5. The aperture is greatest where a high fluid overpressure occurs in the soft layer F. Generally, the aperture decreases from the fracture centre to its tips, and so does the applied fluid overpressure. However, in the soft layer C the aperture is much larger than in the stiffer adjacent layers, and the decrease in aperture in the soft layer I is much more abrupt than in the adjacent layers. This aperture variation would normally encourage channelling of subsequent horizontal fluid flow.

repeated up to the surface, so that the topmost layer A has a Young's modulus of 100 GPa. The hydrofracture is confined and thus cannot propagate into the top and bottom layers, in which the model is fastened. The fluid overpressure is applied along the entire height or dip dimension of the hydrofracture and varies linearly from 10 MPa at the centre to 0 MPa at the fracture tips (Fig. 4).

The greatest aperture is reached where the soft layer F coincides with a high fluid overpressure near to the centre of the hydrofracture at the contact between layers E and F (Fig. 5). The aperture decreases to the fracture tips as the applied fluid overpressure decreases. There is a small difference in aperture between the moderately stiff (B, E, H) and stiff (A, D, G) layers. But the aperture in the soft layers (C, I) is clearly larger than in the adjacent layers.

Discussion

Numerical models are valuable tools to explore complex physical problems that are not tractable with analytical methods. The input parameters, such as the modelled geometry and the mechanical properties, must, however, be chosen very carefully so as to represent the real geological situation. The test implications of the numerical models must also be checked by field observations; for example, as regards natural fracture systems.

In heterogeneous and anisotropic rocks, many, and perhaps most, hydrofractures become arrested, at various depths, at discontinuities or contacts between rocks with different mechanical properties. Hydraulic fracture experiments in petroleum engineering indicate that the vertical propagation of a hydraulic fracture is commonly arrested at contacts between layers, particularly when the layers have strong mechanical and stress contrasts (Charlez 1997, Yew 1997, Economides & Nolte 2000).

Similarly, field observations show that in layered rocks many hydrofractures become arrested at contacts between mechanically different rock layers. Figure 6 shows an arrested joint in Precambrian gneiss with amphibolite layers, exposed in the city of Bergen, West Norway. The joint, which

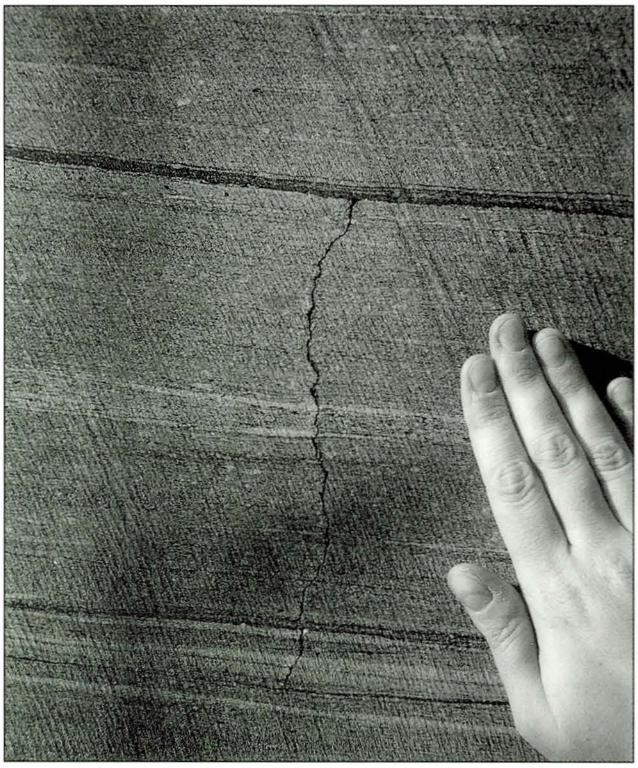

Fig. 6. Arrested open joint in a metamorphic rock, at a road-cut in Bergen, West Norway. Fractures are commonly arrested at contacts between layers of different mechanical properties, such as here at the contact between amphibolite and gneiss. View north-northeast, the hand provides a scale.

may have originated as a hydrofracture, extends from gneiss and ends abruptly at its contact with the amphibolite. There is normally a large difference in the laboratory stiffnesses of gneiss and amphibolite (Hansen et al. 1998, Myrvang 2001). In Norway, the stiffness of amphibolite can vary between 30 GPa and 130 GPa, with the most common values perhaps between 40 GPa and 110 GPa (Hansen et al. 1998). By contrast, the stiffness of gneiss can vary between 10 GPa and 150 GPa, while the most common values are perhaps between 20 GPa and 80 GPa (Hansen et al. 1998). Thus, amphibolite can be either stiffer or softer than gneiss. In cases where the amphibolite was stiffer than the gneiss, the arrest of the fracture at their contact could be attributed to the generally high horizontal compressive stresses that are currently operative in West Norway (Hicks et al. 2000, Myrvang 2001), which would concentrate in the stiff layer. By contrast, if the amphibolite was softer than the gneiss, the arrest of the fracture could be attributed to its being generated as a hydrofracture, with fluid overpressure as the only loading, similar to that in the model in Fig. 3. Which arrest mechanism operates depends not only on the stiffnesses of the rock layers, but also on the (unknown) time of fracture arrest because different stress fields operate at different times.

In igneous rocks, dykes are commonly arrested at contacts between lava flows or between lava flows and layers of pyroclastic rocks. This is seen, for example, for many dyke tips in Tenerife (Canary Islands) and Iceland (Gudmundsson et al. 1999, Marinoni & Gudmundsson 2000). Many arrested dyke tips have also been observed at bedding contacts in sedimentary rocks (Baer 1991). In sedimentary rocks, there are also many joints and mineral veins that are stratabound (restricted to one layer) (Odling et al. 1999, Gillespie et al. 2001). For example, calcite veins are abundant inside limestone layers but become arrested at their contacts with softer marl layers (Gudmundsson & Brenner 2001).

These field observations support the results of the numerical models (Figs. 1-3), that soft layers are effective barriers for propagating hydrofractures. In these models, a hydrofracture would normally propagate through a stiff layer, if it is not a stress barrier. A stress barrier is a layer where the hydrofracture-normal compressive stress is higher than in adjacent layers. Stress barriers are particularly common in mechanically layered rocks subject to compressive stresses; the stiff layers usually take up most of the compressive stress, become highly stressed, and arrest hydrofractures (Gudmundsson 1990, Gudmundsson & Brenner 2001). Such barriers commonly coincide with abrupt changes in Young's moduli, horizontal discontinuities or both (Gudmundsson 1990, Gudmundsson & Brenner 2001), all of which can contribute to the arrest of hydrofractures (Gudmundsson & Brenner 2001).

At this stage, comparison of the results of the numerical models concerning the aperture variation of hydrofractures (Figs. 4-5) with field observations is, however, more difficult. One reason is that the difference in aperture, particularly for small-scale hydrofractures such as mineral veins, is commonly too small to be noticed. Very soft layers are commonly to some extent ductile and sustain little or no tensile stresses. Hydrofractures, such as veins, dykes or joints that enter such layers are thus likely to trigger failure in shear rather than in extension. Thus, a hydrofracture that propagates through such a soft layer is likely to follow an inclined shear fracture rather than go vertically through the layer as an extension fracture. Many fractures also follow existing weaknesses in the host rock (like cooling joints or foliation), in which case parts of their pathways may be inclined. An inclined hydrofracture is normally not perpendicular to the horizontal minimum principal compressive stress, but rather to the (higher) normal stress, and thus becomes thinner. Local changes in the fluid overpressure because of stress changes normal to the fracture, for example due to stress barriers, may also lead to aperture changes.

The aperture (thickness) variation of dykes propagating through mechanically layered rocks in Iceland was observed by Gudmundsson (1984). His results indicated that dyke aperture inside soft pyroclastic rock layers tends to be greater than where the dyke cuts through the centre of a stiff basaltic lava flow. In layered rock masses subject to extension, where stiff layers take up most of the tensile stress, the aperture in the stiff layers may be greater than in the soft layers (Gudmundsson & Brenner 2001). We plan to carry out more field studies to investigate the effects of changing elastic properties of the host rock in layered reservoirs that are explored in the numerical models in this paper.

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